Hamilton-Jacobi Reachability for Spacecraft Collision Avoidance

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Outline

1. **Introduction**
2. Related Works
3. Methodology
4. Results

Image courtesy: SpaceX

Introduction

The commercialization of LEO bring new opportunities, and new concerns:

- Space debris
- Increased spacecraft operations in LEO
- 8000 → 42,000 satellites in LEO
- Industry is looking to scale small spacecraft operations
- Space traffic management very human-in-the-loop today

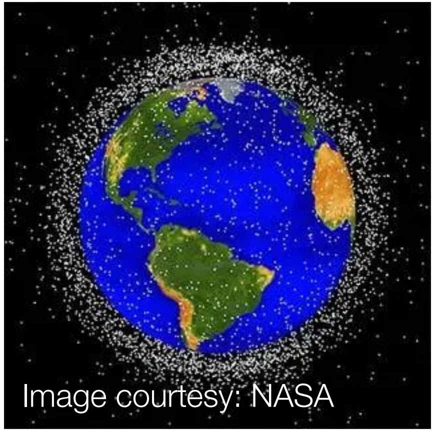
Image courtesy: NASA

Concerns on spacecraft collision avoidance for future large scale constellations like starling. Calls for ways to guarantee collision avoidance in LEO.

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Related Works

Many methods of safety guarantee exist in the literature:

- Control Barrier Functions (CBFs)
- Model-Predictive Shielding
- Simplex Architecture
- **Hamilton-Jacobi Reachability**

HJ Reachability Applications

- Air traffic management (ATC)
- UAVs
- Multi-agent (robotic) coordination
- In-air refueling

New applications for space traffic management where there are no disturbances and dynamics are well known

Pros to using HJ Reachability

- Provable synthesize the safe behaviors of satellites
- Human satellite controllers are unpredictable, model based approaches fail
- Model the safe set of maneuvers given uncertainty
- Can be used to look at real time solvers like CBF later

First step towards thinking about safe maneuvers and guarantees for satellites

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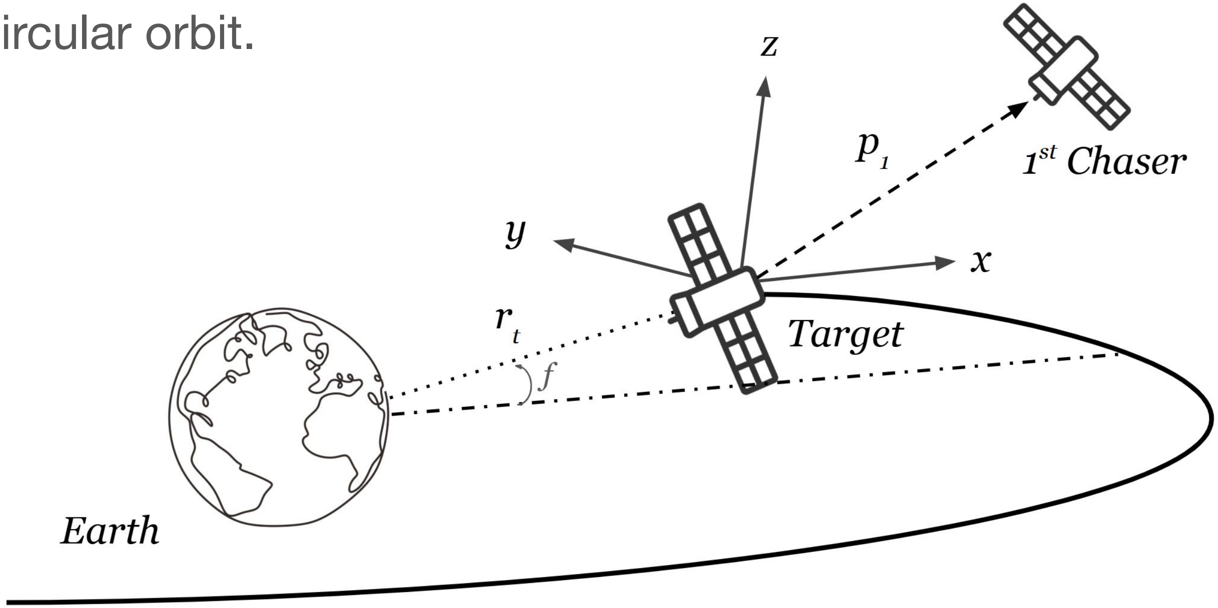
Problem Formulation

Two-satellite collision avoidance scenario in which a controlled satellite (Player 1) operates in proximity to a secondary or uncooperative satellite (Player 2) within the same circular orbit.



Satellite Model

$$m\left(\frac{d^2\mathbf{r}}{dt^2} + 2\boldsymbol{\omega} \times \frac{d\mathbf{r}}{dt} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})\right) = -\frac{Gmm_1}{R_1^3}\mathbf{R}_1 - \frac{Gmm_2}{R_2^3}\mathbf{R}_2$$

Restricted 3-body problem (1)

$$\frac{d^2\mathbf{R}}{dt^2} + 2\boldsymbol{\omega} \times \frac{d\mathbf{R}}{dt} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times (\mathbf{R} + \boldsymbol{r}_1)) = -\frac{\omega^2 r_1^3}{r^3}\mathbf{r}$$

Non-Linear Relative Dynamics (2)

$$\frac{d^2\mathbf{R}}{dt^2} + 2\boldsymbol{\omega} \times \frac{d\mathbf{R}}{dt} = -\omega^2\mathbf{R} + 3\omega^2(i_\xi \cdot \mathbf{R})i_\xi$$

Linearized 3D Vector Equations (3)

$$\begin{cases} \frac{d^2x}{dt^2} + 2\omega\frac{dy}{dt} = 0 \\ \frac{d^2y}{dt^2} - 2\omega\frac{dx}{dt} - 3\omega^2 y = 0 \end{cases}$$

Unforced Planar HCW Dynamics (4)

$$\dot{X} = f(X,u,d) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ -2\omega\dot{y} + u_x + d_x \\ 2\omega\dot{x} + 3\omega^2 y + u_y + d_y \end{bmatrix}$$

State-Space Formulation (5)

Hybrid System and Differential Game

Hybrid System Architecture

- decentralized collision avoidance governed by the discrete set of operational modes $\mathcal{Q} = \{q_{\text{nom}}, q_{\text{eva}}, q_{\text{ret}}\}$
  - these correspond to station-keeping, escape, and trajectory recovery phases
- Autonomous state-dependent guard transitions

$$\begin{aligned} q_{\text{nom}} \to q_{\text{eva}} &: \; X \in \partial\mathcal{G}(\tau), \\ q_{\text{eva}} \to q_{\text{ret}} &: \; X \in \mathcal{X}_{\text{safe}}, \\ q_{\text{ret}} \to q_{\text{nom}} &: \; X \in \mathcal{G}_0^{\text{rec}}, \; |\dot{x}| \le \varepsilon_x, \; |\dot{y}| \le \varepsilon_y \end{aligned}$$

- The boundary of the BRS $\partial\mathcal{G}(\tau)$ serves as the critical switching surface mandating immediate emergency evasion.

Zero-Sum Differential Game Setup

- Player 1 operates not knowing the secondary satellite's intent → worst-case robust guarantees are enforced via an antagonistic differential game setup.
- **Unsafe Target Set** $\mathcal{G}_0$: is the immediate planar collision zone, mathematically bounded using a circular constraint per the FCC minimum orbital separation

$$\mathcal{G}_0 = \{X \in R^4 | x^2 + y^2 \le d_{FCC}^2\}$$

- **Adversarial Disturbance Space** $\mathcal{D}$**:** To natively handle perturbations dynamically, Player 2's action space is closed and deterministically bounded:

$$\mathcal{D} = \{d \in R^2 \mid d_x \in [d_{x,\min}, d_{x,\max}], \; d_y \in [d_{y,\min}, d_{y,\max}]\}$$

HJ Reachability

1. **Optimal Value Function Evaluation**: Evaluated over a finite time horizon T tracking the continuous path trajectory $\xi_{X,T}^{u,d}(t)$; propellant consumption is secondary to immediate vehicle survival → running cost $L(X, u)$ = 0.

$$\phi(X,T) = \inf_{u \in \mathcal{U}} \sup_{d \in \mathcal{D}} \min_{t \in [0,T]} \phi_0(\xi_{X,T}^{u,d}(t))$$

2. **Optimal Control Minimax Hamiltonian**: Synthesizes the global continuous plant behaviors against the spatial gradients of the developing value function $\nabla\phi$

$$\mathcal{H}(X, \nabla\phi) = \min_{u \in \mathcal{U}} \max_{d \in \mathcal{D}} \langle f(X,u,d), \nabla\phi \rangle$$

3. **Hamilton-Jacobi-Isaacs Variational Inequality**: To compute the BRT, the collection of all states that breach $\mathcal{G}_0$ at *any* point along T, the value function is prevented from increasing once a violation occurs, using a variational inequality:

$$\min\left(\frac{\partial\phi}{\partial t} + \mathcal{H}(X,\nabla\phi),\ \phi_0(X) - \phi(X,t)\right) = 0 \quad \text{Initial Condition: } \phi(X,0) = \phi_0(X)$$

4. **Provable Sublevel Safety Boundary**: The terminal output of the backward integration defines the definitive safe/unsafe switching surface for the supervisor automaton; any relative initialization satisfying $X(0) \notin \mathcal{G}(\tau)$ can be mathematically proved to have collision-free control policies regardless of adversarial behaviours

$$\mathcal{G}(\tau) = \{X \in \mathbb{R}^4 | \phi(X, -\tau) \leq 0\}$$

Control Law

1. PD Control Tracking

Continuous thrust inputs applied via tangential (u_x) and radial (u_y) accelerations govern nominal tracking and the trajectory recovery phase. State-feedback laws drive spacecraft towards target positions and target velocities within localized RTN frame

$$u_x = -k_{p,x}(x - x_f) - k_{d,x}(\dot{x} - \dot{x}_f)$$
$$u_y = -k_{p,y}(y - y_f) - k_{d,y}(\dot{y} - \dot{y}_f)$$

Note the integral term, or lack thereof, in the state-feedback laws.

2. Actuator Saturation Constraints

To accurately model physical propulsion system thresholds, symmetric saturation bounds restrict the maximum allowable control authority available to the system:

$$\mathcal{U} = \{(u_x, u_y) \in \mathbb{R}^2 | \cdots \quad u_x \in [-u_{\max}, u_{\max}],\ u_y \in [-u_{\max}, u_{\max}]\}$$

3. Directional Escape Maneuvers

Upon an imminent safety violation causing a discrete transition into the active evasion state q_{eva} the nominal slot target is suspended. The automation selects one of four distinct radial or velocity escape modes determined by relative positioning constraints:

1) **Mode 1**: Player 1 attempts to move to a higher altitude (positive radial) which naturally causes it to drift backward relative to Player 2 (negative tangential).
2) **Mode 2**: Player 1 attempts to move to a lower altitude (negative radial) which naturally causes it to accelerate forward relative to Player 2 (positive tangential).
3) **Mode 3**: Player 1 executes a pure tangential braking maneuver to increase separation distance behind Player 2 without altering its radial altitude.
4) **Mode 4**: Player 1 executes a pure tangential acceleration maneuver to get ahead of Player 2 on the same altitude.

Controller Gain Parameters

Controller Gain Parameters

Parameter	Symbol	Nominal/Recovery	Evasive	Units
Tangential P Gain	$k_{p,x}$	1.0×10^{-4}	5.0×10^{-4}	s^{-2}
Radial P Gain	$k_{p,y}$	1.0×10^{-4}	5.0×10^{-4}	s^{-2}
Tangential D Gain	$k_{d,x}$	2.0×10^{-2}	4.4×10^{-2}	s^{-1}
Radial D Gain	$k_{d,y}$	2.0×10^{-2}	4.4×10^{-2}	s^{-1}

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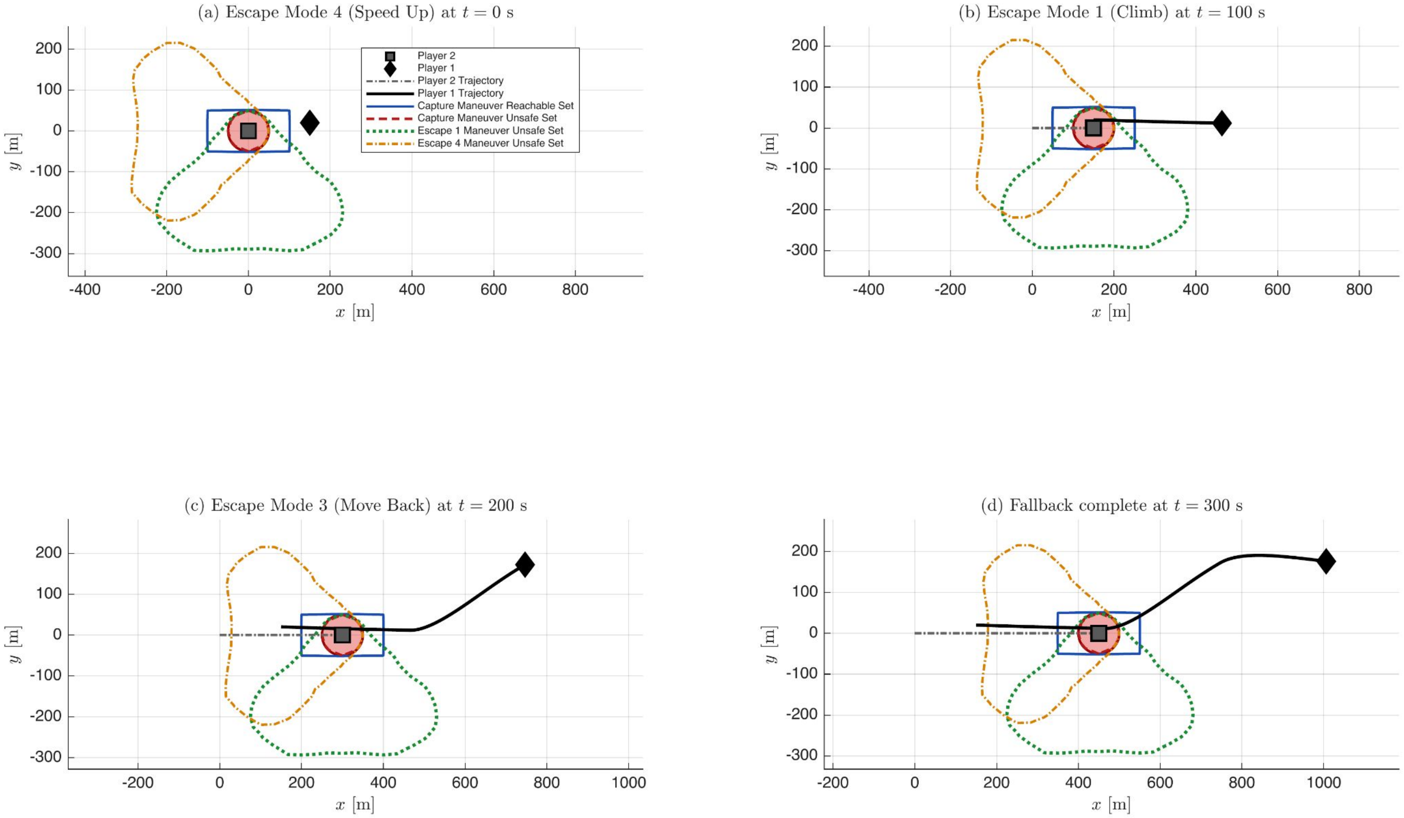
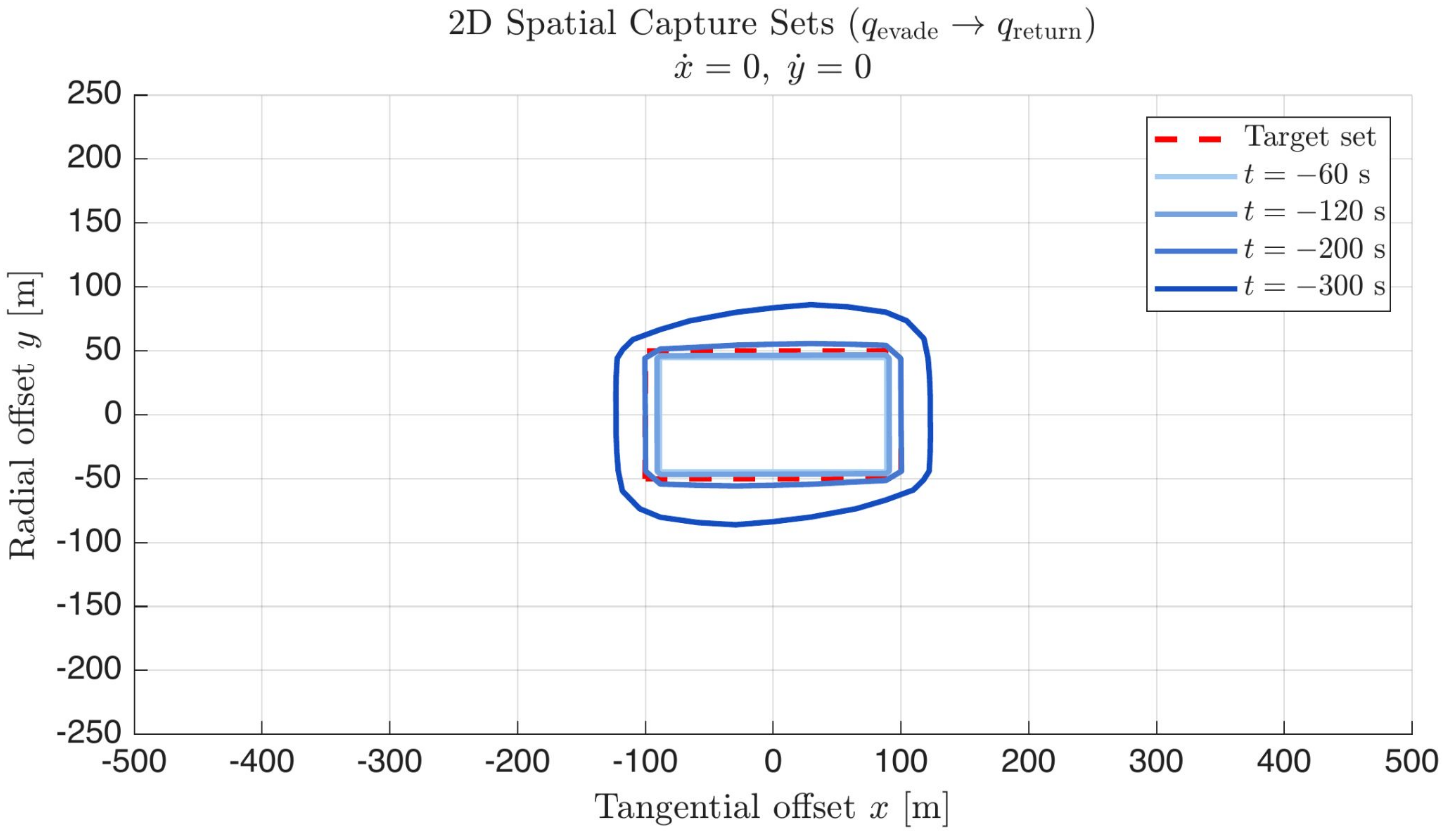
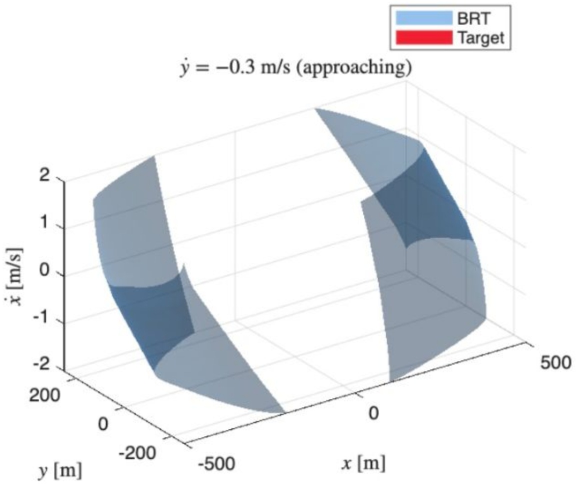
Fallback Scenario Trajectories



Capture Set Calculations



Backwards Reachable Set



Future Work

- Future work will extend the framework to full 3D relative motion to incorporate out-of-plane dynamics, more realistic orbital perturbations and nonlinear orbital effects.

- Additional work will investigate scalable approximations and decomposition techniques to mitigate the computational burden associated with high-dimensional reachable set computation.

- Further studies would emphasize less conservative uncertainty descriptions, such as probabilistic or adaptive disturbance models